

Development of pitch-class set theory in China

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Abstract. This paper mainly studies the development of the "pitch-class set theory" in China after 1980s, and divides this theory into three levels: Forte's set theory, combinatorial set theory and dynamic set theory. The development and research of the set theory in China mainly were mainly focused in two areas: one is the introduction and development of the set theory in China; the other is the further development and research of the set theory in China, which is mainly reflected in the formation of the combinatorial and dynamic set theory.

Keywords: Pitch-Class Theory, Atonity, Allen Forte, Modern Music

Introduction

The theory of pitch-class set is originated from Hauer's Tropen theory, which was then applied to practical works by composers in the twentieth century. The Schönberg's twelve-tone technique is built on the inheritance of Tropen theory, and has a far-reaching impact on subsequent musicians. The pitch-class set theory has formed different theoretical frameworks in the Western world, among which Howard Hanson, George Perle and Teto Bowen's theories were the earliest, and Allen Forte's pitch-class theory was formed after.

As an important music analyzing theory in the twentieth century, the "pitch-class theory" has spread to China for thirty years. Chinese scholars have also made some contributions to the development of the set theory while translating and studying the relevant documents on it.

1. Study on Forte's pitch-class set theory

1.1 Initial introduction to the Forte's pitch-class set theory

As a systematic theory of modern music analysis and practice, the pitch-class set theory was introduced into China in the early 1980s and has been further developed.

Forte's pitch-class set theory was first seen in the article "The Control of Basic Sets over Twelve Tones published by Professor Zheng Yinglie in the first issue of Chinese Musicology in the mid-1980s. This paper is not aimed to explain the set theory, but is a detailed description of the relationship between the sequential linear truncation and sequence (basic set) in structural analysis of twelve sequences. It was the primary stage in exploring the basic relationships of pitch-class set: equality, complementary and inclusion, and put forward a detailed explanation of the nature of the pitch-class set - interval vector.

1.2 Further research and development of Forte's set theory in China

The premise of studying pitch-class set theory is the solution of the set prototype. The logic process in calculating set prototype is quite complicated and abstract, and the method of calculation is also complicated. In this regard, Chinese scholars have conducted a thorough research, which has further developed the logic of calculating set prototype.

Compared with Forte's logic of prototype solution, Chinese scholars are more focused on simple

calculation methods. In terms of solving logic, they mainly take the different intervals of "major" and "minor" as the criteria, and form two kinds of solving logics, namely "major standard" and "minor standard". These two different solutions are mainly discussed in two articles: Simple Method of Calculating Set Prototype and Music Theory in Solution of Set Prototype.

1.1.1 Solving logic of "major standard"

In Forte's theory of pitch-class set, the standard order is to find the minimum value of the first and last two tones in a circular arrangement, that is, the set with n elements has n kinds of arrangement. It is relatively easy to solve a set of tri-tone and quad-tone which has a minor number of tones, while the calculation of a set with a large number of tones is more complicated. In Mr. Luo Zhongrong's Simple Method of Calculating Set Prototype, he firstly elaborated some concepts about the pitch-class set theory, and analyzed Forte's logic of calculating set prototype in detail. At last, he explained his own simple calculating method. The calculating process is also based on the cyclic arrangement, but only concentrates on one line. "In this way, the correct arrangement of the standard order still needs to be screened many times, and the process is rather tedious. In fact, the minimum difference between the first and last two tones of the cyclic arrangement is the maximum between the adjacent two tones in an ordered series of sound."



Figure 1

In Figure 5 of the series of sound $(0, 2, 5, 9, 10)$, the largest difference between adjacent tones (the difference between the second tone and the previous one) is between F-A, with a difference of 4, so that A or F tones can be used as the prefix of the preset standard order: if F tones are used as the prefix of the standard order, then the standard order is the inverse of the set; if A tone is used as the prefix of the standard order, then the standard order is the prototype of the set. If the set has the prefix of F tone, the difference between the first tone and the second tone is 3, or it has the prefix of A tone, then the difference between the first and the second tone is 1. Then take the minimum difference between adjacent two tones in these two different sets, which means that the set with prefix A tone is the standard order of the set. After shifting, the set 5-27 $(0, 1, 3, 5, 8)$ is formed.

1.1.2 Solving logic of "minor standard"

In Forte's pitch-class set theory, the calculation of standard order is also based on a "minor standard", but Forte's own logic is based on the first and last two tones of the circular arrangement, while the Chinese scholar's method of "minor standard" solution is free from the arrangement of a circular sequence. Chinese scholars only compared the interval difference between adjacent two tones on the basis of the ordered arrangement.

Tong Zhongliang's Music Theory in Solution of Set Prototype is a complete exposition of this solution. In this paper, the calculating process of set prototype is divided into three steps: arranging the analyzed musical sounds in sequence, calculating the semitone number between adjacent two tones and finding the minimum interval in the example, arrange a sequential order to one end of the adjacent minor interval start from the minimum interval, and pointing out the half-tone number of the intervals formed by the set of initial tones then marked it with Arabic numerals.

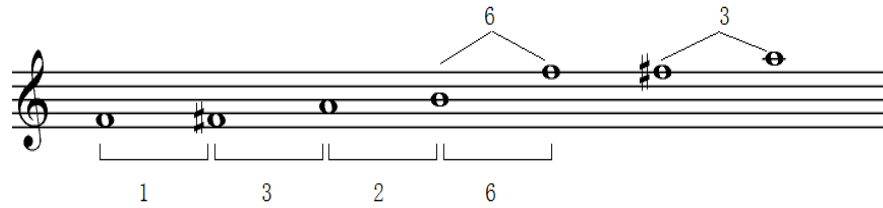


Figure 2

In Figure 6, firstly, the set is arranged by taking the minimum difference between adjacent tones, which is 1; starting from this interval, its left end is a minor third degree and its right end is a tritone, then it is arranged in order to the right end; after shifting the set, the set prototype 4-Z15 (0, 1, 4, 6) can be calculated.

Chinese scholars has simplified the standard order from different perspectives(minor standard and major standard), which not only promotes the further development of Forte's theory of pitch-class set, but also plays an important role in the popularization of the pitch-class theory in China.

2. Research on the combinatoriality of the pitch-class set

In the late 1980s, many scholars in China began to study the combinatorial set theory. The earliest attempt was the book *Serial Composition and Atonality* translated and introduced by Mr. Qin Yuanping. The original work was written by George Perle, an American composer.

Subsequently is Mr. Gao Weijie's *The Combinatoriality of Pitch- Class Set*, which summarizes all the possibilities of the combination of the pitch-class set. It includes 5 two-tone set, 11 three-tone set, 7 four-tone set and 20 six-tone set. In addition, it has proposed a new theory of set free combination and partial combination, which is his further development of the combinatorial set theory. Local matching is relative to overall matching. "Partial matching means that the matching of isomorphic sets only occurs in the range of local sounds (4 to 10 sounds) in 12 sound fields; the matching sets have at least one pair (two sets); the set composed of two or three sounds may also form more matching pairs (e.g., two sound sets can reach five matching sets, three sound sets can reach 3 matching sets). It can reach three sets of matching). The total number of matches is limited with only 54 kinds in total. The five-tone set can not form the whole matching. This is because the seven-tone set is not the divisor of 12. At the same time, the seven-tone set and more than seven-tone set can not form the matching, but the relative partial matching will not be so limited."

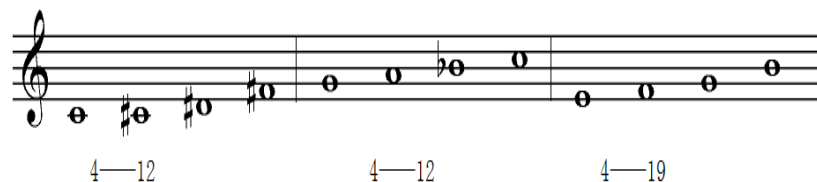


Figure 3

In Figure 7, this twelve-tone sequence can be divided into three four-tone sets. The first two four-tone sets are 4-12, and the last four-tone set is 4-19, which conforms to the general characteristics of partial matching.

In addition to the above research literature, there are many research results in analysis and creation. For example, Mr. Yao Henglu's *Principles of Combinatorial Composing in Three Tones Set-An Analysis of Babbitt's Semi-Simple Variations* is one of the documents on the theoretical study of combinatorial set theory; Mr. Chen Shisen's articles, *Methods and Principles of Writing Complete*

Combinatorial Sequences under the Guidance of Set Table and Theory and Techniques of Set Programming Control has explained some methods and theories of the combinatorial set theory in practice, which has an important guiding significance for Chinese music creation.

3. Research on dynamic set theory--dynamic harmony of atonal Music in the 20th Century

After more than 20 years of exploration and research by Chinese scholars, the theory of dynamic combinatorial set of has achieved fruitful results. Some of them have developed foreign research results, and some are based on new theoretical cornerstones. The representative research achievements include Mr. Gao Weijie's Harmony Mechanics Research - Classification and Quantitative Analysis of Harmony Tension Effect of Altitude Sets, Mr. Zhao Xiaosheng's Pitch-set Techniques - Taiji Composition System, and Mr. Tong Zhongliang's A Brief Introduction to the Series of Modern Harmony Strings.

Zhao Xiaosheng's Pitch-set Techniques - Taiji Composition System is one of the important achievements in the study of the pitch-set theory, and is also an important document in the study of dynamic set theory. As a special topic of Taiji composing system, "Pitch-set Techniques" occupies a considerable space in the study of the dynamic set theory, which fills the blank of the research on the subject of dynamic set in many aspects in China.

In the book Taiji Composition System, there is a formula for calculating the structural strength of a single set of sounds, that is, the number of sounds contained divided by the sum of the total intensity of the various intervals of the set.

In the twelvetone equal temperament, the base rate of the structural strength of the set of any interval (the static dynamic property of the set) is: pure fifth = 1.5, major third = 2.5, minor third = 3, tritone = 3.5, major second = 4, minor second = 4.5.

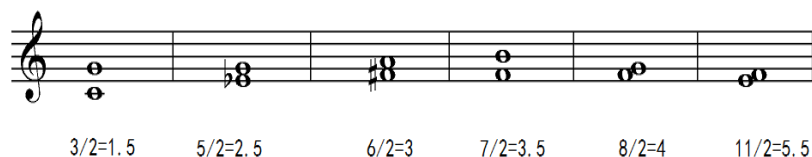


Figure 4

In Figure 8, the numerator 2 is chosen because any interval is a two-tone set, so the two-tone numerator obtained by the above formula is the same, but the denominator is different, which is determined by the order in which each interval appears in the harmonic series. As we all know, the initial intervals in the harmonic series are harmonious and stable. As the overtone is farther away from the pitch and the programmed energy of the pitch decreases, the interval extending to the top of the harmonious series becomes more and more disharmonious, that is, more and more unstable, and the power becomes stronger and stronger. Therefore, for a two tone set, the quantitative calculation of different interval dynamics can be determined according to the interval position in the harmonious series, usually according to the position of interval crown sound in the harmonious series.

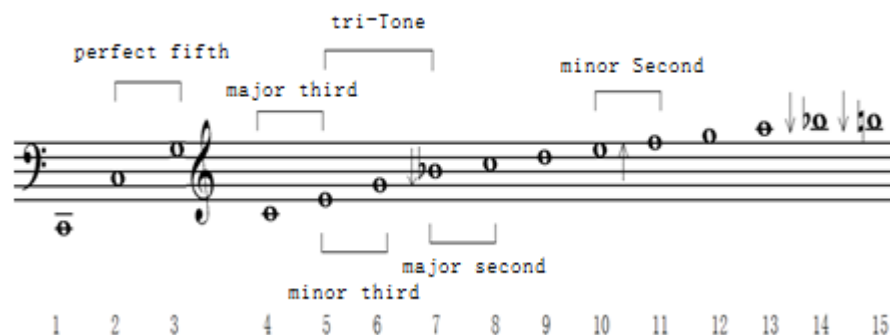


Figure 5

Various intervals are the cells that make up a set. The occupancy of different intervals in interval function determines the dynamic properties of a set.



Figure 6

$$6—15 = \frac{3 \times 5.5 + 2 \times 4 + 3 \times 3.5 + 4 \times 3 + 2 \times 2.5 + 1 \times 1.5}{6} = 8.92$$

$$6—30 = \frac{2 \times 5.5 + 2 \times 4 + 4 \times 3.5 + 2 \times 3 + 2 \times 2.5 + 3 \times 1.5}{6} = 8.09$$

In Figure 6, the prototypes of these two sets are 6-15 and 6-30 respectively. Although they are both six-tone sets, the intensity of each sound structure is different due to the different volume of intervals. Through calculation, the sound structural strength of set 6-15 is 8.92, while that of set 6-30 is 8.10. The difference of sound structure results in the dynamic efficiency of the set.

The article *A Brief Introduction to the Series of Modern Harmony Strings* is a comprehensive compilation of Mr. Tong Zhongliang's special speech on National Academic Symposium on Harmony and Acoustics in Music Institutions of Higher Learning in 1986. In this paper, the study of the set dynamic is based on interval function, but in the specific calculation of the set dynamic, Tong's paper is more focused on the number of specific intervals contained in a sound set. Therefore, it can be said that the main basis of dynamics set theory in the article *A Brief Introduction to the Series of Modern Harmony Strings* is the interval characteristics contained in the sound set. The stability of different intervals is expressed by definite number, such as the definite number of minor second interval is 1, major second interval is 2, minor third interval is 3, major third interval is 4, pure fourth interval is 5, tritone is 6. However, when analyzing the dynamics of a set, the definite number 1 and 5 are the criteria to measure the stability of a set, i.e. using feature quantification. "Although the quantitative analysis of features sacrifices the accuracy of quantitative analysis, but the convenient and simple quantitative analysis just meets the needs of musical ontology characteristics.

Conclusion

Forte's pitch-class set theory has a far-reaching impact on China's music theory. However, in addition to Forte's pitch-class set theory, Chinese scholars have done a lot of research on the set theory, including the "combinatorial set" theory, the "set dynamics" theory and so on.

As one of the most important music analyzing theories in the twentieth century, the theory of pitch-class set has been developing for more than thirty years in China. This not only promotes the development of music analysis in China, but also deepens the concept of modern music creation in China to a certain extent. This paper has collected the data of the development and research status of the pitch-class theory China, hoping to play a role of inducing more excellent works. I hope that experts and colleagues will put forward valuable opinions and hope that this theory will be further developed and improved in the process of modern music in the twenty-first century.

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